

A NONINTERRUPTING METHOD FOR MEASURING
CHARGED PARTICLE BEAM CURRENTS

A THESIS

Presented to
The Faculty of the Graduate Division
by
William Carl Lineberger

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Electrical Engineering

Georgia Institute of Technology
October, 1962

"In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

60
12R

A NONINTERRUPTING METHOD FOR MEASURING
CHARGED PARTICLE BEAM CURRENTS

Approved:

Date approved by Chairman: Oct. 31, 1962

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the valuable contributions to this research made by my thesis advisor, Dr. J. W. Hooper, who suggested this problem and followed its development with great interest. The other members of my thesis reading committee, Dr. E. W. McDaniel and Dr. E. W. Martin, performed a valuable service with their careful review of this manuscript. Finally, I owe a great debt of gratitude to the National Science Foundation, whose fellowship support made this year of graduate study possible.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	ii
LIST OF ILLUSTRATIONS	iv
SUMMARY	v
Chapter	
I. INTRODUCTION	1
II. ANALYSIS OF THE MAGNETOMETER	6
Conditions for Equivalence of Beam and Wire Currents	
Magnetic Field Sampling Current Measurement	
III. EXPERIMENTAL PROCEDURES AND RESULTS	17
Experiments Utilizing a Simulated Beam	
Experiments Utilizing a Chopped Electron Beam	
IV. CONCLUSIONS AND RECOMMENDATIONS	25
APPENDIX I. THE VOLTAGE SENSITIVITY OF HALL GENERATORS	27
APPENDIX II. MEASUREMENTS WITH DUTY CYCLES OTHER THAN	
FIFTY PER CENT	31
BIBLIOGRAPHY	35

LIST OF ILLUSTRATIONS

Figure		Page
1.	Apparatus Used by Whitlock and Hilsum to Measure Charged Particle Beam Currents	4
2.	Schematic Diagram of Basic Circuit to be Analyzed	7
3.	Analysis of a Square Wave Wire Current	9
4.	Approximate Flux and Voltage Waves Induced by a Chopped Ion Beam	12
5.	Schematic Diagram of Experimental Apparatus Utilizing a Simulated Beam	18
6.	Response of the Measurement Circuit to a Simulated Beam . . .	20
7.	Application of the Measurement Device to a Typical Atomic Collisions Experiment	24
8.	Beam Current with Arbitrary Duty Cycle	32

SUMMARY

Charged particle beams are employed in many experiments in atomic collisions research. The beam currents are normally measured by collecting the beam particles in a Faraday cup and measuring the resultant current from the cup to ground. In many experiments, however, it would be desirable to measure the beam current without interrupting it; the development of such a device is the object of this research.

It appears that any noninterrupting measurement scheme must utilize the magnetic field associated with the moving charged particles. Using an indium antimonide Hall effect probe as a magnetic field detector, Whitlock and Hilsum were able to detect a 10^{-5} ampere electron beam. This device is not sufficiently sensitive for profitable application in many beam experiments. As a part of this study, a bibliography of methods of increasing the sensitivity of Hall effect generators has been compiled.

The primary emphasis in this research, however, has been placed on the noninterrupting measurement of charged particle beams consisting of a series of periodic pulses. The pulsed beam is allowed to pass through a toroid on which is wound a coil. It is shown that, if the distance traveled by a beam particle during one pulsing period is large compared with the toroid dimensions, then the voltage induced in the coil consists of a series of impulses, the time integral of each impulse being proportional to the beam current. It is further shown that this

voltage is equal to that induced in the coil by a one turn loop wound on the toroid, and carrying a current of the same wave shape and magnitude as that of the pulsed beam.

It is possible to use this second loop to induce a voltage opposing that induced by the beam, and then employ the magnetometer as a null indicator. At the null, the current in the loop must equal the beam current; thus measurement of the null current provides the noninterrupting measurement of the beam current. If, in addition, a capacitor is placed across the coil and adjusted to resonate with it at the chopping frequency, the sensitivity of the magnetometer is increased by a factor equal to the Q of the resonant circuit. This fact may be seen from a Fourier expansion of the induced voltage waves.

Using this resonated magnetometer with the null loop and a 50 per cent duty cycle beam, currents of 3×10^{-7} amperes peak have been measured with an accuracy of $\pm 10^{-8}$ amperes. While this measurement employed a wire simulating the beam, the analytical results were also substantiated with an actual electron beam. It is shown that duty cycles other than 50 per cent may also be employed. This device, which was developed in this research, affords an improvement in sensitivity of nearly two orders of magnitude over the only other noninterrupting beam measurement method found in the literature.

CHAPTER I

INTRODUCTION

Charged particle beams play an important role in many experiments in atomic collisions research. These beam currents are normally measured by collecting the beam particles in a Faraday cup and measuring the resultant current from the cup to ground. The only objection to the use of a Faraday cup lies in the obvious fact that the cup physically interrupts the beam; it is thus impossible to obtain continuous beam current data at more than one location in the beam path.

In a typical experiment¹ in atomic collisions, a monoenergetic beam of H^+ ions passes through a target gas and is collected in a Faraday cup. A set of collector plates parallel to the beam produces a uniform electric field in the interaction region. The field is sufficiently strong to sweep out the charged particles resulting from ionization of the target gas, and yet does not significantly deflect the high energy H^+ beam. A knowledge of the gas pressure, the possible reactions, the target gas dimensions, and the currents to the collector plates and Faraday cup enables one to determine the gross ionization cross section. It would be desirable, however, to measure in addition the H^+ ion current just prior to its entry into the reaction region. Such a measurement is not possible with a Faraday cup.

¹J. W. Hooper, E. W. McDaniel, D. W. Martin, and D. S. Harmer, Phys. Rev. 121, 1123 (1961).

When both projectile and target are unstable, as in the case of electrons incident on lithium ions, the measurement problem is even more severe. It is necessary to produce beams of both species, and then examine the changes in beam composition resulting from the intersection of the two beams. In an experiment such as this, the number density of residual gas molecules in the intersection region is unavoidably quite significant compared to the beam particle densities. The interaction of the beams with the residual gas may completely mask the mutual interaction of the beams. This difficulty may be overcome by pulsing (chopping) both beams and examining only the chopping frequency component of the current resulting from the interaction. Such a procedure has been devised in a novel experiment by Dolder, Harrison, and Thonemann.² Here again it would be valuable to be able to measure these chopped beam currents prior to their intersection. In both these experiments, the ion beam current was between 10^{-5} and 10^{-7} amperes. While the first type of experiment does not require the use of a chopped beam, such a beam can equally well be used.

This thesis is concerned with the development of a method for measuring chopped charged particle beam currents as low as 10^{-7} amperes without interrupting the beam. It would appear that any feasible approach to this problem would involve measurement of the magnetic field associated with the moving particles. This magnetic field is, of course, directly proportional to the beam current. The magnitude of such a field

²K. T. Dolder, M. F. A. Harrison, and P. C. Thonemann, Proc. Roy. Soc. A264, 367 (1961).

is quite small compared to the earth's magnetic field, which is on the order of one gauss in air. For example, the magnitude of the magnetic field of a 10^{-7} ampere beam at a distance of one centimeter from the beam in free space is roughly 1.5×10^{-8} gauss. There are two ways in which the magnetic field may be utilized: (1) the field may be measured directly or (2) the field may be used to induce a second current which is directly proportional to the beam current.

A technique similar to (2) has been in use in the power industry for many years, namely the use of current transformers to measure line currents without the inconvenience and the dangers associated with placing the measuring equipment directly in the high voltage line. Such measurements, however, have never been extended to small currents.

At first sight, it would appear that a device utilizing the well-known Hall effect is ideally suited for (1), since, at a constant control current through a Hall probe, the Hall voltage is directly proportional to the transverse magnetic field. Using the Hall effect in indium antimonide, Whitlock and Hilsum³ were able to detect a 10^{-5} ampere electron beam. Figure 1 depicts the apparatus which they employed. An indium antimonide Hall generator is placed in an air gap built into a mumetal ring surrounding the electron beam. The presence of the ring greatly increases the magnetic field appearing across the Hall generator. The dc electron beam is equivalent to a one turn coil wound on the mumetal ring. The current balance loops serve as a one turn coil whose magnetic field opposes that of the electron beam. If the current through

³W. S. Whitlock and C. Hilsum, Nature 185, 302 (1960).

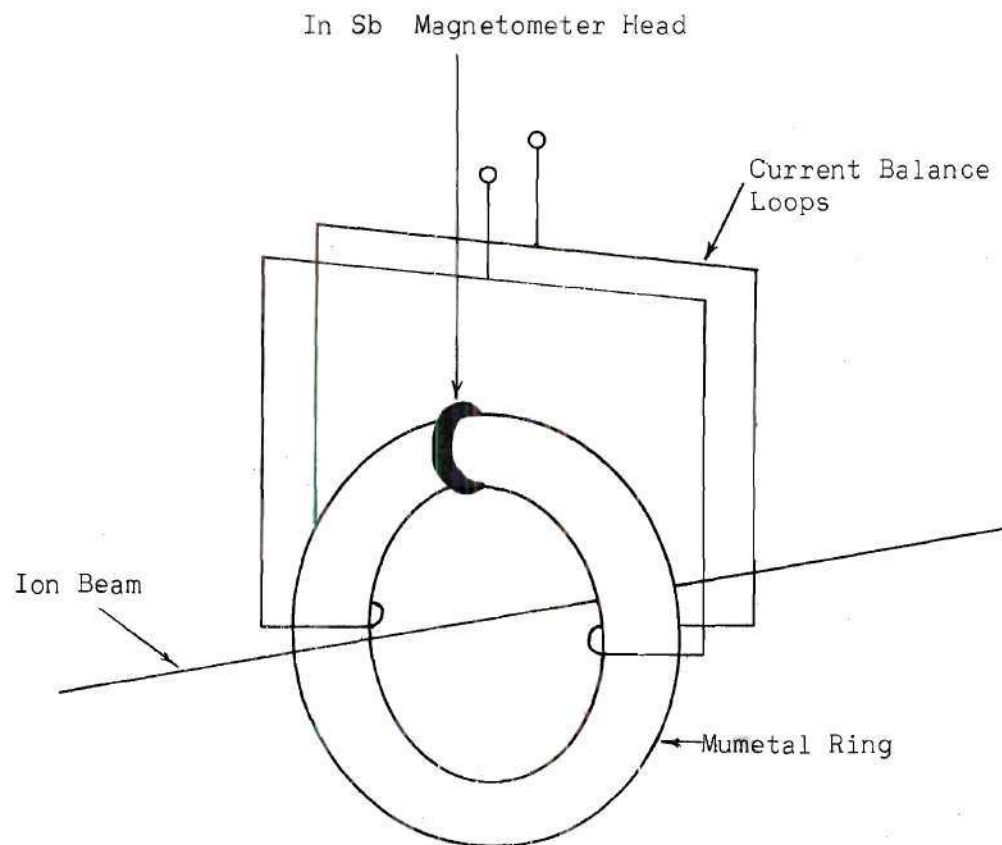


Figure 1. Apparatus Used by Whitlock and Hilsum to Measure Charged Particle Beam Currents

the balance loops is adjusted for zero Hall voltage across the crystal, then the balance loop current should equal the beam current. While direct currents can be measured easily, this device was also used to measure chopped beam currents by nulling only the chopping frequency component of the Hall voltage.

Although the apparatus of Whitlock and Hilsum is not sufficiently sensitive for profitable employment in atomic collisions research, it appears to be the only noninterrupting beam current measuring device reported in the literature. A drastic improvement in the sensitivity of this device appears to be a difficult task. A literature search has been made in order to find ways to increase the sensitivity of Hall generators; the results of this search are presented in Appendix I.

The method of measurement employed in this study utilizes a voltage induced by the chopped beam (Method 2).

In this approach, the voltage induced by a chopped beam in a coil wound on a high permeability ring serves as a measure of the beam current. One of the principal advantages of this method is the increased magnetic field intensity obtained by eliminating the air gap from the magnetic circuit. Chapter II will consist of a mathematical development of the measuring device, while Chapter III will present experimental procedures and results.

CHAPTER II

ANALYSIS OF THE MAGNETOMETER

The basic circuit which we shall analyze is shown in Figure 2. The radius of the toroid is R meters, its cross section is A meters², and its relative permeability is μ_r . The coil has N turns, and the permeability of free space is μ_0 henrys per meter. The charged particle beam is chopped into segments $D/2$ meters long, and $1/T$ pulses per second pass a given point in the beam path.

It is obvious that such an arrangement will result in some sort of voltage being induced in the coil. If that voltage were sufficiently large, it would be possible to determine an induced voltage versus beam current calibration curve and use the device to perform a noninterrupting beam current measurement. A serious disadvantage to this method is that a change in the magnetic state of the toroid may change its permeability, and hence the calibration curve. The principal advantage of the null scheme used by Whitlock and Hilsum is that a calibration curve is not necessary and the resulting measurement is unaffected by changes in the magnetic state of the toroid. While it would be very desirable to utilize this nulling scheme, a simple application is not possible unless the voltage induced in the coil by the chopped beam can be duplicated by a square wave current in the null loop. The conditions for the equivalence of beam and wire currents will be determined in the next section.

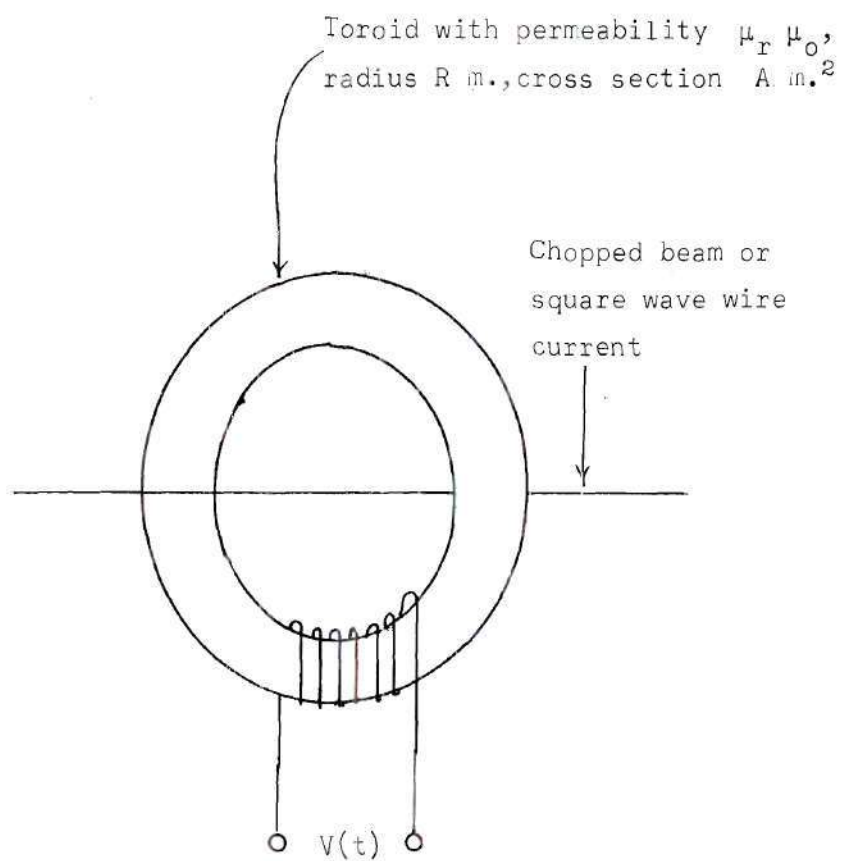


Figure 2. Schematic Diagram of Basic
Circuit to be Analyzed

Conditions for Equivalence of Beam and Wire Currents

It must be recognized that there is a basic difference between an idealized square wave current on a wire of infinite extent and a chopped ion beam insofar as their magnetic effects are concerned. With the square wave wire current, electrons throughout the entire length of the wire are drifting during the "on" time, while the net motion is zero at any point during the "off" period. With a chopped beam, the particles move in discrete bundles, separated by a void. Since the magnetic field at a point is determined by the current throughout the entire beam or wire, the magnetic effects of the beam and current are decidedly different, in general. We shall now show that if the beam particle velocity is so large that the distance D traveled by a particle in one chopping period is much greater than the physical dimensions of the toroid, then the beam and wire currents may be considered to be identical in the following sense. Consider the voltage induced in the coil of Figure 2 by a square wave wire current or a chopped ion beam. We shall say that the beam and wire are equivalent if and only if equality of the beam and wire currents implies equality of the fundamental frequency components of the Fourier series expansions of the voltage induced in the coil by the respective currents.

Voltage Induced by a Square Wave Wire Current

Let us now compute the fundamental component of the voltage induced in the coil of Figure 2 by the idealized square wave current having peak value I_m amperes shown in Figure 3a. If the permeability of the toroid is much greater than that of free space, the long wire may be considered

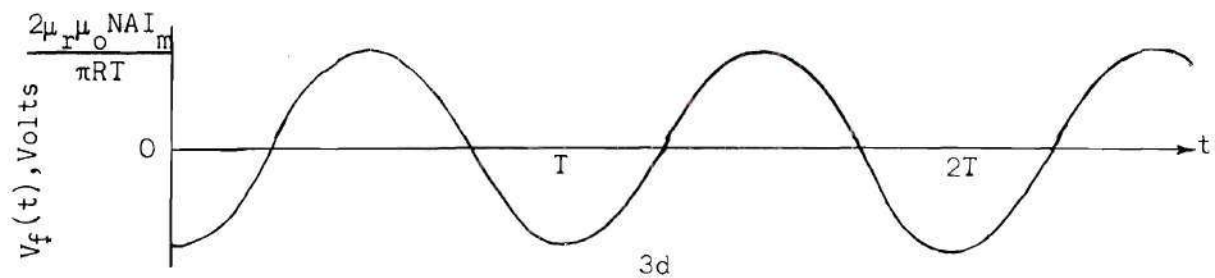
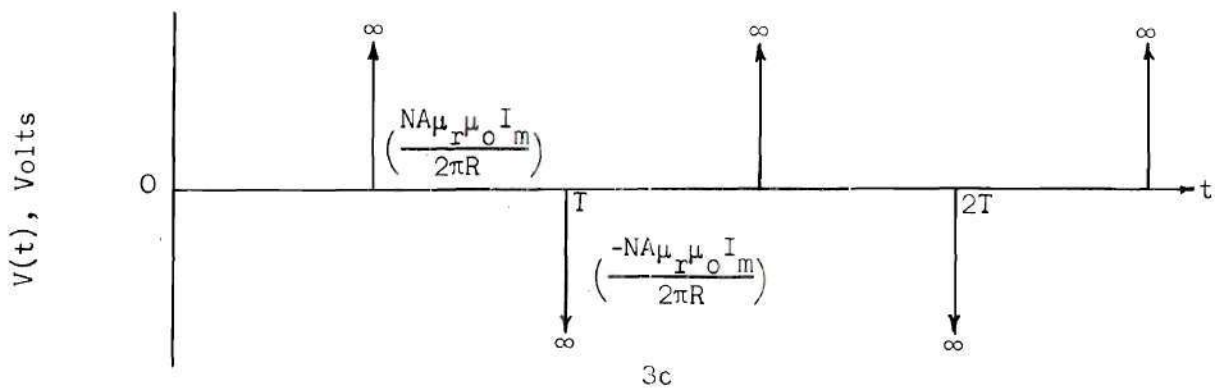
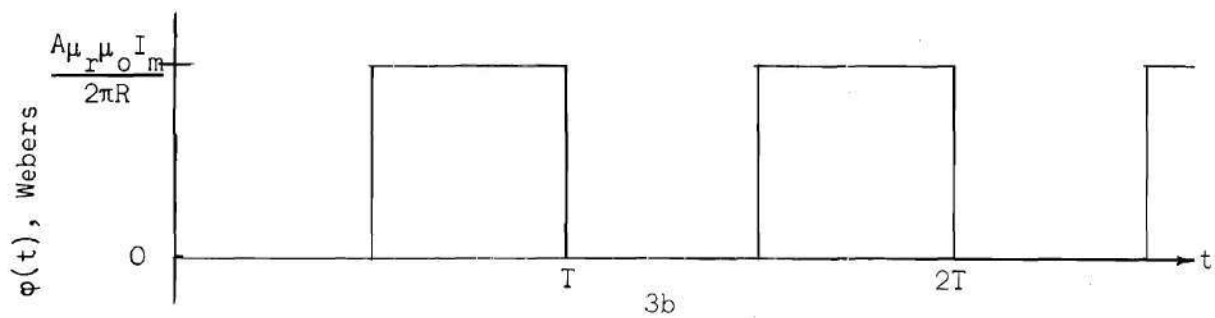
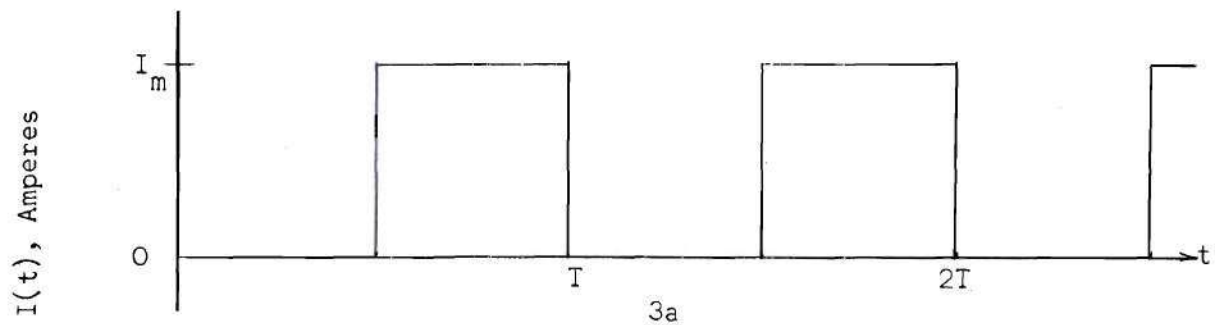


Figure 3. Analysis of a Square Wave Wire Current

to be a one turn coil wound on the toroid, with negligible leakage flux. In such a situation the resultant flux in the toroid is readily seen to be that shown in Figure 3b.

$$\phi(t) = \frac{A \mu_r \mu_o}{2\pi R} I(t) . \quad (1)$$

The induced voltage $V(t)$ is also seen to be

$$V(t) = N \frac{d\phi(t)}{dt} \quad (2)$$

which is the impulse train shown in Figure 3c. The quantity in parentheses beside each impulse represents the area under each impulse. A Fourier analysis of $V(t)$ yields

$$V(t) = - \frac{2 \mu_r \mu_o N A I_m}{\pi R T} \sum_{k=0}^{\infty} \cos \frac{2\pi t}{T} (2k + 1) \quad (3)$$

and the fundamental component is given by

$$V_f(t) = - \frac{2 \mu_r \mu_o N A I_m}{\pi R T} \cos \frac{2\pi t}{T} . \quad (4)$$

The assumption of an idealized square wave current makes the magnitudes of all of the harmonics equal.

Voltage Induced by a Chopped Charged Particle Beam

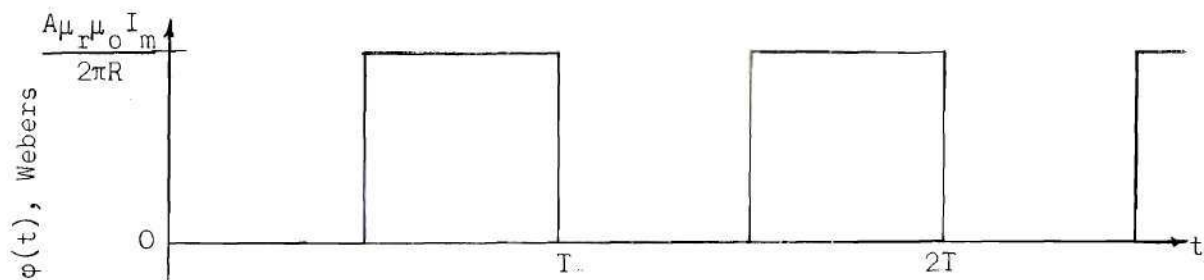
We will now analyze the chopped beam subject to the restriction that the distance D traveled by the beam during one period (T seconds) is much greater than any of the dimensions of the toroid. This requirement is met in many cases of experimental interest; a 5 keV Li^+ beam,

for example, chopped at 1000 cps travels about 2×10^4 cm. in one period, while the toroid dimensions are on the order of one cm.

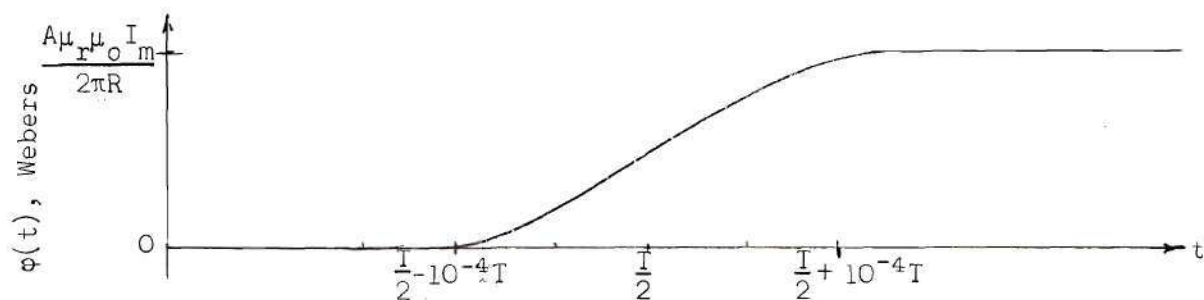
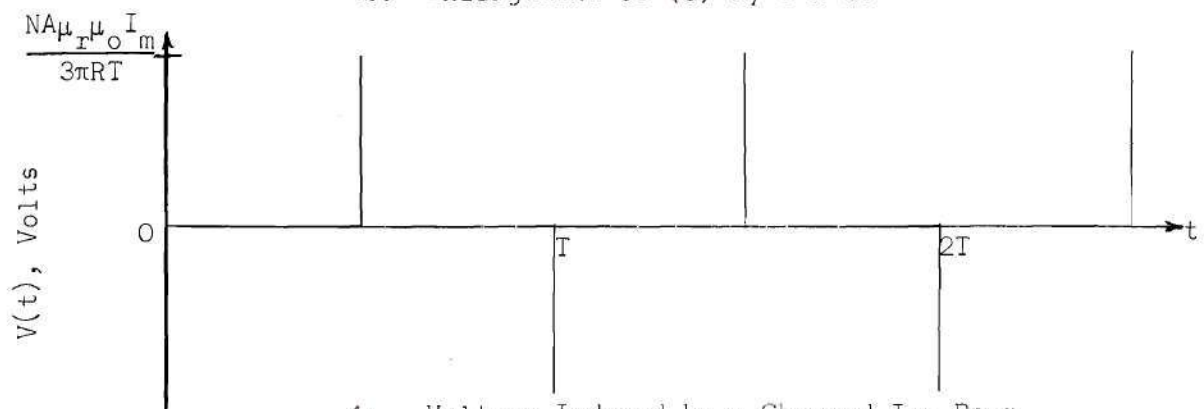
An exact analysis of the flux produced by such a beam would be at best a tedious process. It may be determined approximately by dividing the toroid cross section into a series of rectangles. The flux in any rectangle is assumed to be equal to the flux density at the center of the rectangle times the area of the rectangle. The flux density at the center of each rectangle can be determined analytically if we assume that the permeability is everywhere μ_0 . The flux in these rectangles is plotted as a function of time and the total time varying flux wave is the graphical sum of all of the individual contributions. Multiplication of this result by μ_r approximately accounts for the permeability of the toroid. This procedure is simply a graphical integration using Simpson's approximation to the integral

$$\phi(t) = \int \int B(x,y,t) \, dx \, dy . \quad (5)$$

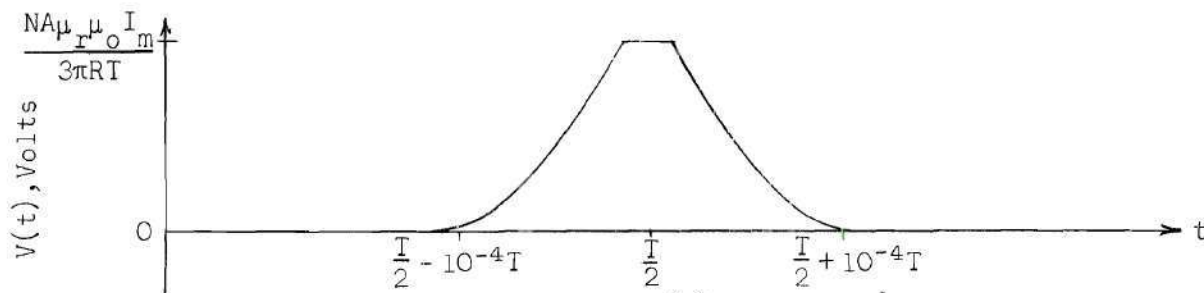
Such an analysis was carried out for a toroid of radius 1 cm. and cross section 1 cm.^2 , and a beam of 5 kev Li^+ ions chopped at 1000 cps ($D = 2 \times 10^4$ cm.). The flux wave was obtained by breaking the 1 cm.^2 toroid cross section into five rectangular strips, as explained. This flux wave is depicted in Figure 4a, while an expanded view of the flux during the changing part of the cycle is shown in Figure 4b. We see from the latter graph that the flux in the toroid changes from less than one per cent of ϕ_{max} to greater than 99 per cent of ϕ_{max} in 2×10^{-4} T seconds. The induced voltage pictured in Figure 4c and the



4a. Flux Wave Produced by a Chopped Ion Beam

4b. Enlargement of (a) by 5×10^8 

4c. Voltage Induced by a Chopped Ion Beam

4d. Enlargement of (c) by 5×10^8 Figure 4. Approximate Flux and Voltage Waves Induced by a Chopped Ion Beam ($D/R = 2 \times 10^4$)

enlarged view shown in Figure 4d were obtained by graphical differentiation of the flux wave.

This case which we have solved approximately serves to justify the following assumptions concerning the flux produced by the beam:

(1) The beam current produces a continuous, time differentiable flux wave in the toroid.

(2) There exists an interval $(nT/2 - p, nT/2 + p)$ around each voltage pulse outside of which the induced voltage is essentially zero.

(3) $2p \ll T$, the chopping period.

Upon integration of (2) we obtain

$$\left| \int_{\frac{nT}{2}-p}^{\frac{nT}{2}+p} V(t) dt \right| = N \left| \Phi\left(\frac{nT}{2} + p\right) - \Phi\left(\frac{nT}{2} - p\right) \right| \quad (6)$$

$$\stackrel{\circ}{=} N \Phi_{\max} = \frac{NA \mu_r \mu_o I_m}{2\pi R}.$$

The induced voltage may be considered to be any arbitrary function $V(t)$ satisfying the above requirements. A standard Fourier analysis of $V(t)$ yields

$$V(t) = \sum_{k=0}^{\infty} a_{2k+1} \cos\left(\frac{2\pi(2k+1)t}{T}\right) \quad (7)$$

where

$$a_k = \frac{2}{T} \int_{-p}^{T-p} V(t) \cos\left(\frac{2\pi kt}{T}\right) dt. \quad (8)$$

In particular,

$$\begin{aligned}
 a_1 &= \frac{2}{T} \int_{-p}^{T-p} V(t) \cos\left(\frac{2\pi t}{T}\right) dt \\
 &\doteq \frac{2}{T} \int_{-p}^p V(t) dt - \frac{2}{T} \int_{\frac{T}{2}-p}^{\frac{T}{2}+p} V(t) dt \\
 &\doteq - \frac{4 \phi_{\max}}{T}.
 \end{aligned} \tag{9}$$

Thus we find

$$V_f(t) \doteq - \frac{2 \mu_r \mu_o N A I_m}{\pi R I} \cos \frac{2\pi t}{T} \tag{10}$$

which is exactly the expression obtained for the square wave wire current in Equation (4), provided the beam chopping frequency is the same as the square wave frequency. We have thus obtained the very important result that, with suitable restrictions, the beam and wire currents are equivalent, in the sense employed in this thesis.

For the idealized square wave wire current, all of the nonzero components of the Fourier expansion of $V(t)$ were equal in magnitude to the fundamental frequency component. The higher harmonics of the beam induced voltage are also equal in magnitude to the fundamental component, provided

$$2 kp \ll T \tag{11}$$

where k is the order of the harmonic in question. The equivalence of the beam and wire is due to the fact that the sharpness of the beam induced voltage pulse allows one to consider it as an impulse. Approximations of this type are frequently made in order to simplify the analysis of sampled data control systems.

It is very important to note that we have established equivalence of the beam and wire currents without requiring exact knowledge of the waveshape of the beam induced voltage. The beam need not be perfectly chopped in order to arrive at Equation (10), provided the irregularities in the beam are confined to a region near the chopping region. The equivalence shown above would be of no value if it applied only to perfectly chopped beams, since such beams are not encountered in practice.

Magnetic Field Sampling Current Measurement

In the previous section we have shown that, properly restricted, equal beam and wire currents induce identical voltages in the circuit of Figure 2. It would thus be possible to add a null loop (such as that used by Whitlock and Hilsum) to this circuit, introduce an oscilloscope across the coil as a null indicator, and thereby make noninterrupting beam current measurements. There are two disadvantages to making the measurements in this manner. First, there is no significant increase in sensitivity over the device of Whitlock and Hilsum, and secondly, the winding capacitance of the coil has been neglected. The high frequency components of the voltage pulses may lead to "ringing" at the self-resonant frequency of the coil, and thus alter the output.

This "ringing" may be put to advantageous use if a capacitor is placed across the coil and adjusted to resonate with the inductor at $1/T$ cps. Under this condition, the voltage $V(t)$ is induced in an RLC series circuit* resonant at the fundamental frequency component of the

*This circuit is actually a very complicated circuit with distributed parameters, but if the capacitance of the external capacitor is much larger than the winding capacitance of the coil, then the circuit may be accurately represented by an RLC series circuit.

Fourier series expansion of $V(t)$. Thus if $V(t)$ is given by Equation (3) or Equation (10), the magnitude of the fundamental frequency voltage across the capacitor is given by

$$|V_c| = |V_f| Q_o = \frac{2 \mu_r \mu_o N A I_m Q_o}{\pi R T} \quad (12)$$

while the magnitude of the k^{th} harmonic is*

$$|V_{c_k}| = \frac{|V_f|}{k^2} = \frac{2 \mu_r \mu_o N A I_m}{\pi R T k^2}, \quad k = 3, 5, 7, \dots \quad (13)$$

The lowest harmonic is the third, which is smaller in magnitude than V_c by a factor of $1/9Q_o$. For a high Q circuit ($Q_o \geq 100$), only the fundamental frequency component of the capacitor voltage is significant. The use of the capacitor has increased the sensitivity of the null detecting coil by a factor of Q_o .

While the analysis carried out in this chapter dealt with a beam with a 50 per cent duty cycle, other duty cycles may also be employed. It would also be possible to make current measurements with the RLC circuit resonated at the third, fifth, or higher harmonics of $V_f(t)$. These matters are considered in more detail in Appendix II.

*This same relation is also approximately true for the voltage induced by the chopped beam, provided $(2k + 1) 2p \ll T$.

CHAPTER III

EXPERIMENTAL PROCEDURES AND RESULTS

The analysis carried out in the previous chapter shows that, for our measurement purposes, the chopped beam and square wave wire current are interchangeable. This result may be used to simplify an experimental investigation of the current measuring device. By substituting a wire for the chopped beam, the difficulties associated with producing a beam (vacuum equipment, circuitry, etc.) are circumvented. In the first portion of this chapter we shall deal with this simulated measurement problem, while the remainder shall concern experiments with an actual electron beam.

Experiments Utilizing a Simulated Beam

Figure 5 is a schematic diagram of the apparatus used to simulate the beam current measurement. The amplifier is a Hewlett Packard Model 400D vacuum tube voltmeter, the output of which is connected to the vertical input of a Hewlett Packard Model 120A oscilloscope. The square wave generator is a Tektronix Type 105. Several toroidal cores have been employed. For a typical core $R = 1$ cm., $\mu_r = 250$ (estimated), $A = 1$ cm.², and $N = 600$ turns. This coil resonates with a .1 μ fd capacitor at 2.35 kc.

In order to test the validity of Equation (11), this circuit was first operated with the null loop opened. With a "beam" current flowing,

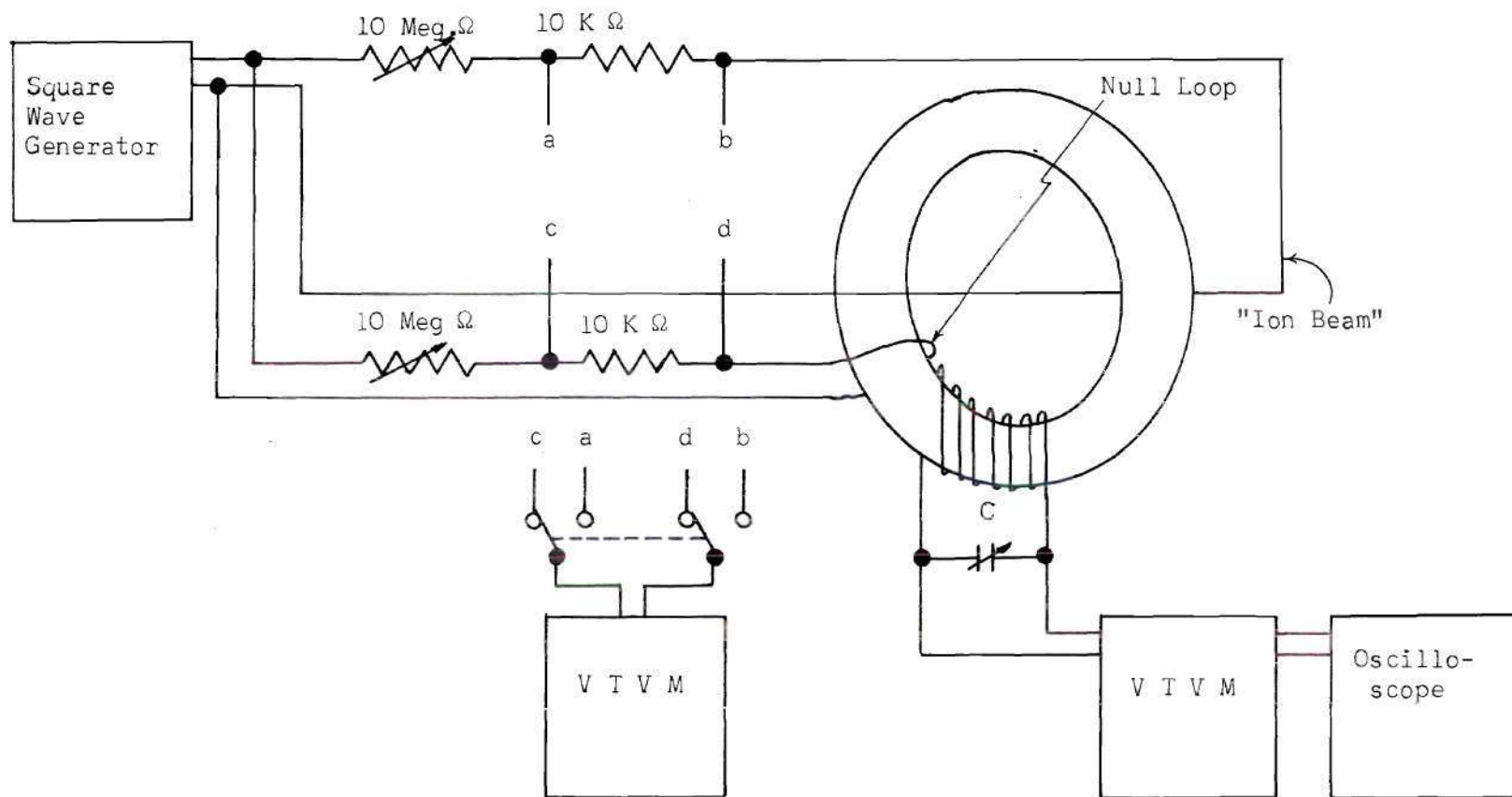


Figure 5. Schematic Diagram of Experimental Apparatus Utilizing a Simulated Beam

the square wave generator frequency was varied until its frequency coincided with the magnetometer resonant frequency. This condition is accompanied by a sudden rise in voltage at the oscilloscope. The output voltage is then plotted as a function of "beam" current. Such a plot is shown in Figure 6. The plot is a straight line passing through the origin, and its slope is approximately that given by Equation (12). Thus the analysis appears to be correct.

With the null loop reconnected, the unknown "beam" current is determined by adjusting the current in the null loop for a null on the oscilloscope. Since the sign of the net current through the toroid depends on whether the null loop current is greater or less than the beam current, the null voltage changes sign as it passes through the null. Once the null has been found, the unknown "beam" current, which is now equal to the null loop current, is determined by measuring the potential difference between points c and d. In this experimental apparatus, the potential difference between points a and b serves to determine the accuracy of the current measurement. Using the coil described previously, a null can readily be obtained for a "beam" current of 3×10^{-7} amperes peak. The accuracy of the measurement is $\pm 10^{-8}$ amperes at this current level. The error is apparently a result of amplifier noise masking the exact location of the null. At higher currents, the "beam" and null loop currents agree as closely as the vacuum tube voltmeter can be read. In the initial measurements, it was found that an instability in the null voltage displayed on the oscilloscope made exact location of the null difficult at low currents. This

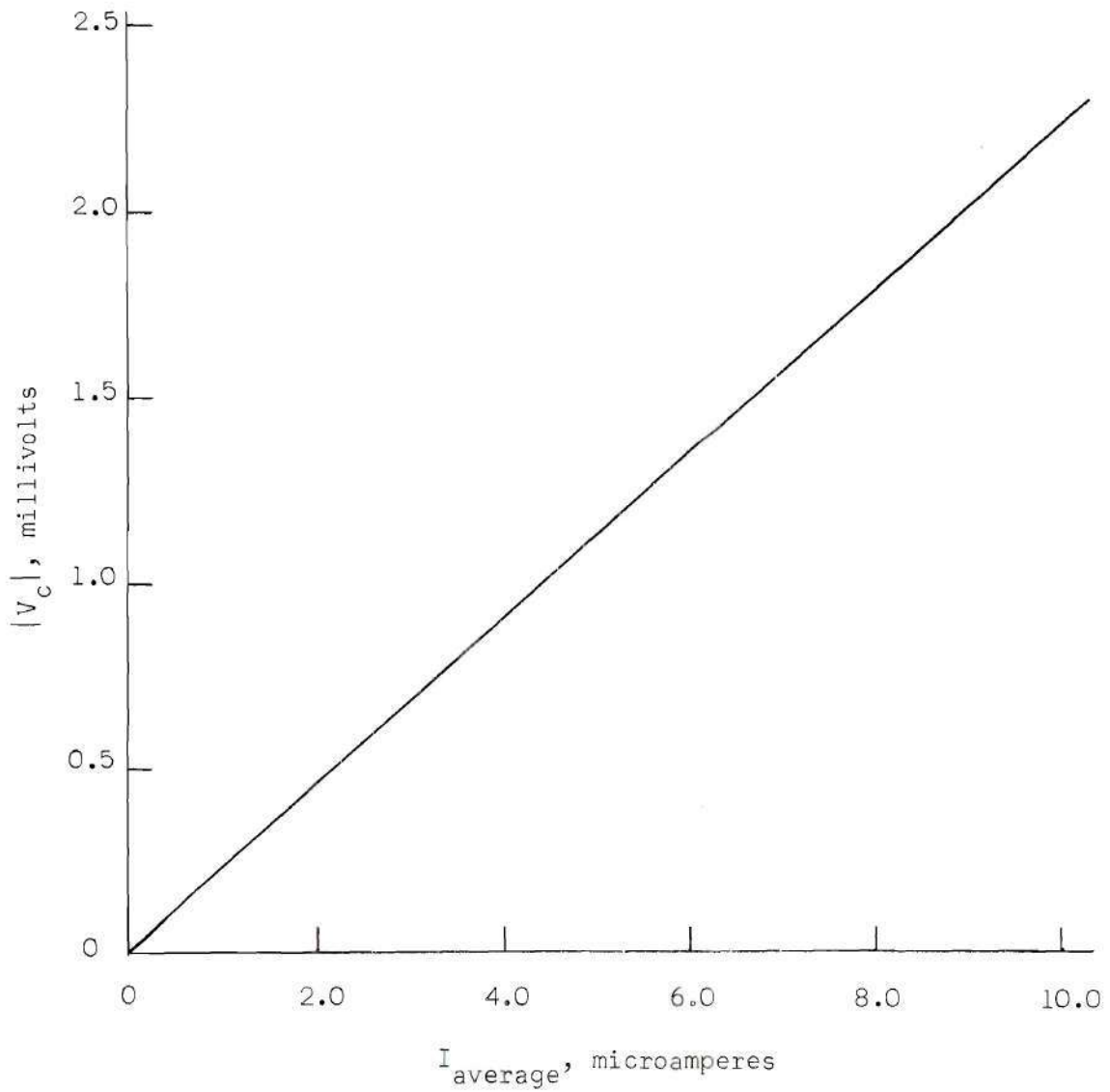


Figure 6. Response of the Measurement Circuit to a Simulated Beam

difficulty was eliminated by the use of line isolation transformers for the two vacuum tube voltmeters.

Experiments Utilizing a Chopped Electron Beam

The experiment just described proves the validity of the measuring technique described in this thesis, provided the square wave wire current is equivalent to a chopped beam. Thus, a demonstration of this equivalence should be the object of any experiment dealing with an actual beam.

The work of Whitlock and Hilsum offers one verification of this equivalence, since the authors claim to have successfully used a null loop technique to measure a chopped electron beam current. If the beam and null loop flux waves were not of very similar shape and magnitude, the null measurement could not have been successful.

In order to verify this equivalence further, a chopped electron beam experiment was carried out by the author. The electron source used was the electron gun from a 3RP1 cathode ray tube. The gun was removed from the tube and placed in a vacuum system, along with a magnetometer and a Faraday cup. The three pieces were optically aligned. As anticipated, the electron source, being of the oxide cathode type, was affected adversely by the exposure to air which occurred during the transfer to the new vacuum system. The electron beam current dropped from 300 μa before transfer to very nearly zero afterward. In an attempt to reactivate the cathode partially, the filament was allowed to operate at 7.0 volts (instead of the normal 6.3 volts) for 15 minutes, with no accelerating voltages applied. At this time, the accelerating and focussing voltages were applied. The beam current slowly rose to 10^{-7} amperes. When

this current had stabilized, the filament voltage was raised in steps to 9.9 volts; the beam current increased correspondingly to $20\text{ }\mu\text{a}$. Deterioration of the gun in the few ensuing hours of operation necessitated further increases in filament voltage to 11.5 volts.

The beam was chopped by applying a 40 volt peak square wave voltage between the cathode and the first grid. The electron energy was 2 kev, and the beam was chopped at 2.35 kc, the resonant frequency of the RLC series circuit. The null loop current was derived directly from the chopping signal, since the distance from the electron gun to the toroid was sufficiently small that the voltage induced in the toroidal coil was essentially in phase with the chopping voltage. The electron mean free path at the operating pressure ($< 10^{-5}\text{ mm.Hg}$) was much greater than the length of the beam path.

The electron source was not capable of producing chopped beam currents greater than $10\text{ }\mu\text{a}$, and source difficulties prevented meaningful measurements at currents less than $7.5\text{ }\mu\text{a}$. In this current range, data were taken for a plot like that shown in Figure 6. In this range the voltage induced in the measuring circuit by the chopped beam equalled that induced in the circuit by a square wave wire current, as determined in a separate experiment. This result indicates equality of the beam and wire currents. Also in this current range, null loop measurements of the chopped beam current were successfully made. While such a limited range of verification of the equality of beam and wire currents is not completely satisfying, it should be recognized that the problems encountered concerned the electron source, and not the measuring

device. When this range of verification is considered together with the work of Whitlock and Hilsum, however, it would appear that the experimental equivalence of beam and wire currents is well established.

Figure 7 is a schematic diagram showing how this measuring device might be employed in an atomic collisions experiment. It should be noted that the signal required for producing the null loop current is taken from the beam chopping plates. Such a null current will be "in phase" with the chopped beam current provided that the distance from the toroid to the chopping plates is small compared with the distance that the beam travels in one chopping period. If this condition does not hold, a variable phase delay network may be inserted between the chopping plates and the null loop to provide a null current of the proper phase. The required phase delay may be computed from the particle velocity (v), the chopping frequency (f), and the distance from the chopping plates to the toroid (d) by use of the relation

$$\theta = \frac{2\pi fd}{v} \quad (14)$$

where θ is the required phase delay, in radians.

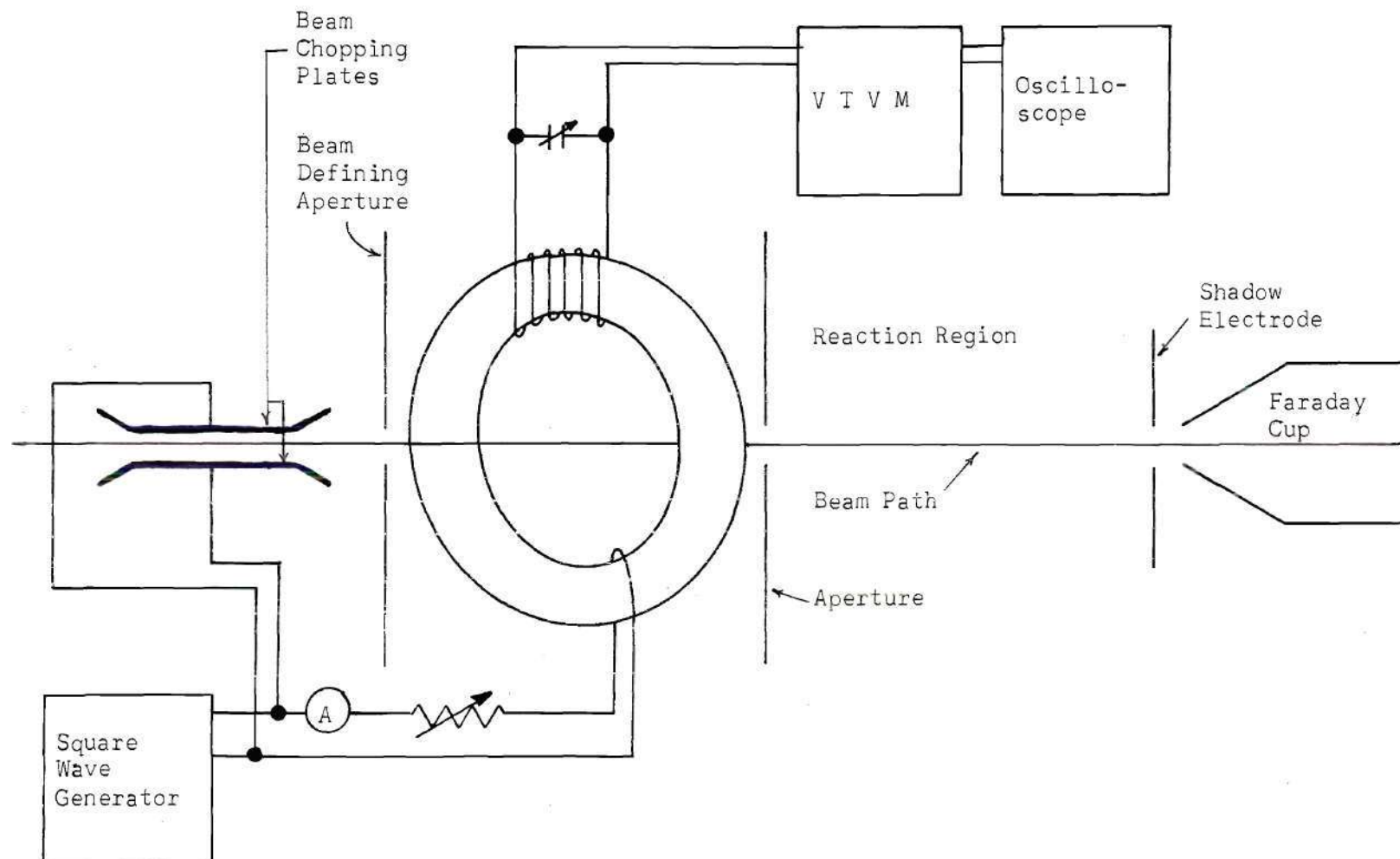


Figure 7. Application of the Measurement Device to a Typical Atomic Collisions Experiment

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

It has been demonstrated that a chopped charged particle beam passing through a toroid induces a series of voltage pulses in a coil wound on the toroid, the magnitude of which voltage is a linear function of the beam current. It has been further shown that if the distance traveled by a particle during one chopping period is much greater than the dimensions of the toroid, the voltage induced in the coil by the chopped beam is very nearly equal to that induced in the coil by a one turn loop wound on the toroid and carrying a square wave current, the magnitude of which is equal to the beam current.

In a device which is apparently original in this research, a resonated coil wound on a toroid is employed in a nulling scheme to permit noninterrupting measurements of chopped charged particle beam currents as small as 3×10^{-7} amperes peak with an accuracy of $\pm 10^{-8}$ amperes. This device is nearly two orders of magnitude more sensitive than the only other noninterrupting device found in the literature, the Hall effect magnetometer of Whitlock and Hilsum. The null scheme used in the noninterrupting current measurement enables the measurements to be independent of changes in the magnetic state of the toroid. An analysis of the induced voltage for an arbitrary duty cycle indicates that noninterrupting measurements may be readily made for duty cycles other than 50 per cent. This analysis has been verified for a duty cycle of 41 per cent.

In the author's opinion, the smallest current measured in this research is not a fundamental limitation or even a practical limitation. A detailed consideration of optimum toroid size and cross section, the number of turns of the toroid, the wire size, and the size of the resonating capacitor should result in significantly lower current measurements. It would also be easily possible to increase the sensitivity by employing toroids possessing higher permeability. The problem of various duty cycles for the chopped beam has been dealt with experimentally for only two cases, and the analytic investigation is relatively brief. This device should find valuable applications in many atomic collisions experiments.

APPENDIX I

THE VOLTAGE SENSITIVITY OF HALL GENERATORS

In this section methods for increasing the sensitivity of the Hall effect magnetometer of Whitlock and Hilsun³ will be considered. The Hall voltage developed by a rectangular Hall generator is given by the well-known expression

$$V_h = \frac{k_h I_c B}{d} f\left(\frac{L}{W}\right) \quad (15)$$

where d is the thickness of the generator, L is the length of the generator in the direction of the control current (I_c), and W is the width of the generator, across which face the Hall voltage appears. The applied magnetic field B is normal to the L - W face of the generator. The quantity k_h is called the Hall coefficient; its value depends on the generator material, and, in general, upon the temperature of the generator. The function $f(L/W)$ is a geometrical form factor which is near its maximum value of unity for $L/W \geq 2.5$.

If, in the magnetic circuit shown in Figure 1, the reluctance of the air gap is much greater than that of the high permeability ring, then the flux density in the air gap is given by

$$B = \frac{\mu_o I_b}{g} \quad (16)$$

where g is the length of the air gap, and I_b is the beam current.

Upon substitution of this latter expression in Equation (15), we obtain

$$V_h = \left[\frac{k_h I_c \mu_o}{dg} f\left(\frac{L}{W}\right) \right] I_b. \quad (17)$$

The voltage sensitivity of the Hall generator is defined here to be the Hall voltage per unit beam current, with no load placed across the Hall terminals.

$$S = \frac{V_h}{I_b} = \frac{k_h I_c \mu_o}{dg} f\left(\frac{L}{W}\right) \quad (18)$$

Thus the problem of detecting smaller beam currents is equivalent to the problem of increasing S . A few remarks on the nature of this problem now follow.

It is presently believed⁴ that indium antimonide (In Sb), which has the highest electron mobility of any present material, is the best choice of material for probe construction, although some authors⁵ argue for other materials. For this application the strong temperature dependence of k_h in In Sb is not detrimental; In Sb is preferred over indium arsenide, whose Hall coefficient is smaller but much less temperature sensitive at room temperature.

The magnetometer of Whitlock and Hilsum employed an In Sb crystal 0.025 cm. thick in a 0.040 cm. air gap. The maximum value of I_c (not

⁴See, for example, C. Hilsum and A. C. Rose-Innes, Semiconducting III-V Compounds, Pergamon Press, Ltd., London, 1961. pp. 209-10;

C. Hilsum, Brit. J. Appl. Phys. 12, 85 (1961).

⁵V. V. Galavanov, Soviet Physics-Solid State 2, 55 (1960);
M. J. O. Strutt, Electronic and Radio Engineer 36, 2 (1959).

specified) is determined by the maximum power that the crystal can dissipate without reaching temperatures injurious to the crystal. Thus a decrease in the thickness of the crystal requires a decrease in the maximum value of I_c . Despite this requirement, the best way to increase the sensitivity appears to be to reduce the thickness of the crystal and the length of the air gap. The reason for this assertion is twofold; first, we see that the decrease in the product dg is substantially greater than the corresponding required decrease in I_c , and, secondly, the decrease in thickness increases the surface to volume ratio of the crystal and thus increases the maximum allowed power dissipation per unit volume. Recent improvements^{6,7} in the techniques of depositing thin films of In Sb have made possible the use of In Sb Hall elements which are only 1.6×10^{-4} cm. thick.

The maximum value of I_c can be further increased by the use of a pulsed control current, which allows larger peak control currents to be employed while keeping the average power dissipation within the allowed limits. Using a control current duty cycle of less than one per cent, Shirer⁸ has found that the peak control current may be safely increased 10 to 20 times above the direct current maximum value. Wieder⁶ has utilized pulse techniques together with a thin film Hall generator to produce a generator of high sensitivity.

⁶H. H. Wieder, J. Appl. Phys. 33, 1278 (1962).

⁷K. G. Gunther, Z. Naturforsch 13a, 1081 (1958).

⁸D. L. Shirer, Rev. Sci. Instr. 31, 1000 (1960).

The use of a Hall generator similar to that described by Wieder in the magnetometer of Whitlock and Hilsum should significantly increase the sensitivity of the Hall effect beam current detector. Such a device has not as yet been reported in the literature. For additional references to applications of the Hall effect, the reader is referred to the excellent bibliography of Hall effect publications recently published by Beckman Instruments, Inc.⁹ The book by Putley¹⁰ treats the Hall effect and other transport phenomena in detail.

⁹"Hall Facts," available from Beckman Instruments, Inc., Helipot Division, Fullerton, California.

¹⁰E. H. Putley, The Hall Effect and Related Phenomena, Butterworth and Co., London, 1960.

APPENDIX II

MEASUREMENTS WITH DUTY CYCLES OTHER THAN FIFTY PER CENT

In Chapter II, a chopped charged particle beam was analyzed for the special case of a 50 per cent duty cycle. It will now be shown that the magnetometer may be used to measure chopped beams having other duty cycles.

Consider a pulsed beam having period T seconds, and pulse duration τ seconds. The pulses occur at $T_1 + nT$ seconds, and the peak current is I_m amperes. Such a beam current is shown in Figure 8. This beam will be equivalent (in the sense employed in this thesis) to a similarly pulsed wire current provided the distance traveled by a beam particle in τ seconds ($T - \tau$ seconds, if $T - \tau > \tau$) is much greater than the dimensions of the toroid. This result is a trivial generalization of the analysis of Chapter II. The equality of beam and wire indicate that a nulling technique will again be feasible. The only remaining problem is to determine whether or not the induced voltage is sufficiently large to permit measurements at the desired beam current levels. We shall resolve this problem by comparing the magnitude of the induced voltage to the magnitude of the induced voltage in the 50 per cent duty cycle case.

When the beam and wire are equivalent, the induced voltage is given by:

$$V(t) = \phi_{\max} [u_0(t - T_1) - u_0(t - T_1 - \tau) + u_0(t - T - T_1) - \dots] \quad (19)$$

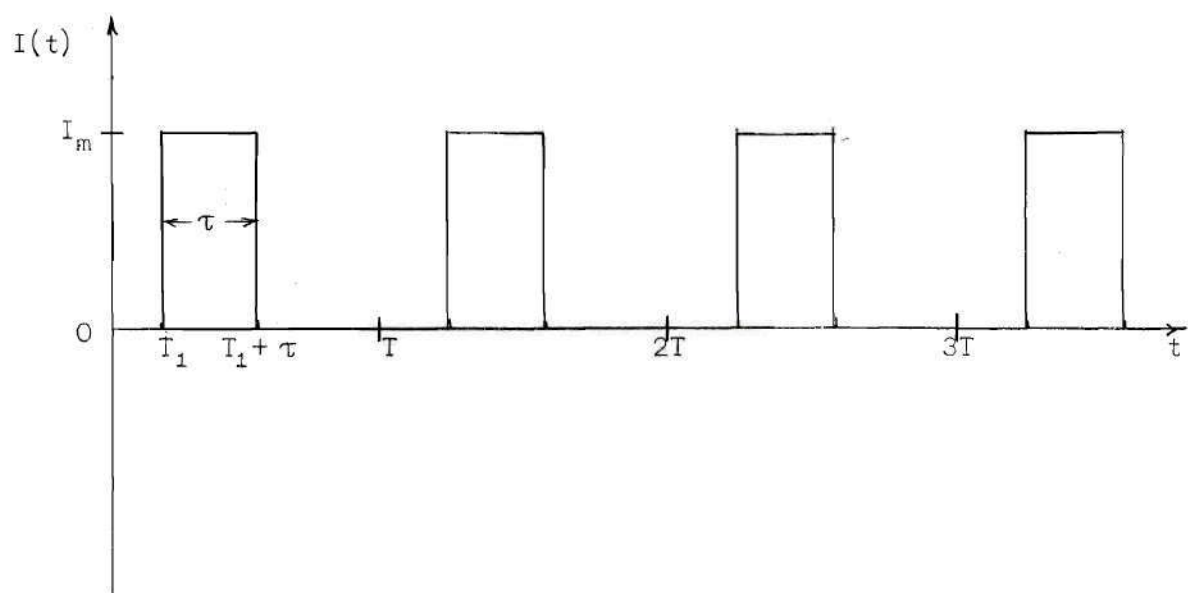


Figure 8. Beam Current with Arbitrary Duty Cycle

where $u_0(t - b)$ is a unit impulse occurring at $t = b$, and

$$\phi_{\max} = \frac{NA \mu_r \mu_o I_m}{2\pi R} \quad (20)$$

The Fourier expansion of $V(t)$ is given by

$$V(t) = \sum_{k=1}^{\infty} \left[a_k \cos \frac{2\pi kt}{T} + b_k \sin \frac{2\pi kt}{T} \right] \quad (21)$$

where

$$a_k = \frac{2}{T} \int_0^T V(t) \cos \frac{2\pi kt}{T} dt \quad (22)$$

and

$$b_k = \frac{2}{T} \int_0^T V(t) \sin \frac{2\pi kt}{T} dt \quad (23)$$

After performing the indicated integration, the fundamental frequency components are given by

$$a_1 = \frac{2 \phi_{\max}}{T} \left[\cos \frac{2\pi T_1}{T} - \cos \frac{2\pi(T_1 + \tau)}{T} \right] \quad (24)$$

and

$$b_1 = \frac{2 \phi_{\max}}{T} \left[\sin \frac{2\pi T_1}{T} - \sin \frac{2\pi(T_1 + \tau)}{T} \right] \quad (25)$$

The magnitude of the resultant fundamental frequency signal is

$$|V_f| = \sqrt{a_1^2 + b_1^2} \quad (26)$$

For $T_1 = \tau = T/2$, the beam current is exactly the 50 per cent duty cycle current analyzed in Chapter II for which

$$|V_f(50)| = \frac{4 \phi_{\max}}{T} . \quad (27)$$

Thus the ratio $\left| \frac{V_f(50)}{V_f} \right|$ is a measure of the decrease in sensitivity resulting from the change in duty cycle.

As a numerical example, let us consider a pulsed beam for which $\tau = .41T$ and $T_1 = 0$. Upon substitution of these values in Equations (24) and (25), we find that

$$\begin{aligned} a_1 &= \frac{-2 \phi_{\max}}{T} 1.844 \\ b_1 &= \frac{2 \phi_{\max}}{T} 0.464 \end{aligned} \quad (28)$$

and

$$|V_f(41)| = \frac{2 \phi_{\max}}{T} (1.844^2 + .464^2)^{\frac{1}{2}} = \frac{2 \phi_{\max}}{T} 1.9 . \quad (29)$$

Therefore

$$\left| \frac{V_f(50)}{V_f(41)} \right| = \frac{\frac{2 \phi_{\max}}{T} 2.0}{\frac{2 \phi_{\max}}{T} 1.9} = 1.05 . \quad (30)$$

Thus if the minimum detectable 50 per cent duty cycle current is 1×10^{-7} amperes, then the minimum detectable 41 per cent duty cycle beam current is 1.05×10^{-7} amperes. The validity of this particular example has been shown with the simulated beam. The computed and experimentally observed values of $\left| \frac{V_f(50)}{V_f(41)} \right|$ agreed within two per cent.

BIBLIOGRAPHY

- Dolder, K. T., M. F. A. Harrison, and P. C. Thonemann, Proceedings of the Royal Society (London) A264, 367 (1961).
- Galavanov, V. V., Soviet Physics-Solid State 2, 55 (1960).
- Gunther, K. G., Zeitschrift für Naturforschung 13a, 1081 (1958).
- Hilsum, C., British Journal of Applied Physics 12, 85 (1961).
- Hilsum, C., and A. C. Rose-Innes, Semiconducting III-V Compounds, Pergamon Press, Ltd., London, 1961.
- Hooper, J. W., E. W. McDaniel, D. W. Martin, and D. S. Harper, The Physical Review 121, 1123 (1961).
- Putley, E. H., The Hall Effect and Related Phenomena, Butterworth and Co., London, 1960.
- Shirer, D. L., The Review of Scientific Instruments 31, 1000 (1960).
- Strutt, M. J. O., Electronic and Radio Engineer 36, 2 (1959).
- Whitlock, W. S., and C. Hilsum, Nature 185, 302 (1960).
- Weider, H. H., Journal of Applied Physics 33, 1278 (1962).